Version 2

MUONIC HYDROGEN GROUND STATE HYPERFINE SPLITTING *

R.N.Faustov[†]

Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, 117333 Moscow, Russia

A.P.Martynenko[‡]

Samara State University, Theoretical Physics Department, Pavlov Street 1, 443011 Samara, Russia

Corrections of orders α^5, α^6 are calculated in the hyperfine splitting of the muonic hydrogen ground state. The nuclear structure effects are taken into account in the one- and two-loop Feynman amplitudes by means of the proton electromagnetic form factors. The modification of the hyperfine splitting part of the Breit potential due to the electron vacuum polarization is considered. Total numerical value of the $1S$-state hyperfine splitting 182.638 meV in the μp can play the role of proper estimation for the corresponding experiment with the accuracy 30 ppm.

PACS numbers: 31.30.Jv, 12.20.Ds, 32.10.Fn

I. INTRODUCTION

The study of the energy levels of simple atomic systems (muonium, positronium, hydrogen atom, muonic hydrogen and others) with high precision plays significant role for the check of the Standard Model and the bound state theory with very high accuracy. The two-particle bound states represent important tool for the exactitude the values of fundamental physical constants (the fine structure constant, the electron and muon masses, the proton charge radius etc.) [1]. The observation of thin effects in low energy physics of simple atoms can be considered as necessary supplement to the construction of large particle colliders for deep penetration to the structure of elementary particles and search of new fundamental interactions. Such atomic experiments can improve our knowledge about elementary particle interactions on small distances what may be reached only at very high energies [2].

The effects of strong interactions play essential role in the energy spectrum of the muonic hydrogen just as electronic hydrogen. On one hand, they are connected with two electromagnetic proton form factors (electric G_E and magnetic G_M) describing the distributions of the

* Talk presented at the 11 th Lomonosov Conference on Elementary Particle Physics, Moscow State University, August 2003

† faustov@theory.sinp.msu.ru

‡ apm@physik.hu-berlin.de

electric charge and magnetic moment. In the Lamb shift case the main contribution to the energy spectrum of order $(Z\alpha)^4$ is determined by the proton charge radius r_p which appeares as differential parameter of these distributions. So, the comparison of the experimental data and theoretical value for the Lamb shift obtained with the corrections of high order over α gives the effective approach to obtain more reliable value of the r_p. The measurement of the $2P-2S$ Lamb shift in μp with the precision 30 ppm allows to obtain the value of the proton charge radius which is an order of the magnitude better in the comparison with different methods [3]. The calculation of the nuclear structure corrections in the hyperfine splitting of the energy levels (see [4, 5, 6, 7, 8, 9, 10, 11]) can be done only on the basis of whole proton electromagnetic form factors. Last experimental measurements of the form factors G_E and G_M were carried out in Mainz 20 years ago [12].

On the other one, important contribution of strong interactions to the hydrogen spectrum is connected with the proton polarizability [13, 14, 15, 16]. It appeares already in the one-loop amplitudes of the muon (electron) proton electromagnetic interaction when different baryonic resonances can be produced in the intermediate states as a result of the virtual Compton scattering on the proton. Exact calculation of such effect can be done by means of experimental data and theoretical models for the polarized nucleon structure functions. The proton structure and polarizability effects lead to main theoretical uncertainty in the expressions for different energy levels including the hyperfine splitting of the hydrogen ground state:

$$\Delta E^{HFS}_{theor} = E^F\left(1+\delta^{QED}+\delta^{str}+\delta^{pol}+\delta^{HVP}\right),\quad E^F=\frac{8}{3}\alpha^4\frac{\mu_P m_1^2 m_2^2}{(m_1+m_2)^3}, \tag{1}$$

where μ_p is the proton magnetic moment in nuclear magnetons, m_1 is the muon mass, m_2 is the proton mass, δ^{QED} represents the QED contribution, δ^{HVP} is the contribution of hadronic vacuum polarization (HVP), the corrections δ^{str} and δ^{pol} are the proton structure and polarizability contributions. The expression (1) is valid both for the muonic and electronic hydrogen but the exact value of these corrections is essentially different for such atoms. The ground state hydrogen hyperfine splitting measurement was made many years ago with very high accuracy [17]:

$$\Delta\nu^{HFS}_{exp}(ep) = 1\ 420\ 405.751\ 766\ 7(9)\ kHz. \tag{2}$$

Existing difference between the theory and experiment without accounting the proton polarizability contribution can be expressed as follows [18]:

$$\frac{\Delta E^{HFS}_{theor}(e\ p)-\Delta E^{exp}_{HFS}(e\ p)}{E^F(e\ p)} = -4.5(1.1)\times 10^{-6}, \tag{3}$$

This quantity contains one of the main uncertainties connected with inaccuracies of the proton form factor determination. Dominant part of the one-loop proton structure correction is defined by the following expression (the Zemach correction) [4]:

$$\Delta E_Z = E^F\frac{2\mu\alpha}{\pi^2}\int\frac{d\mathbf{p}}{(\mathbf{p}^2+W^2)^2}\left[\frac{G_E(-\mathbf{p}^2)G_M(-\mathbf{p}^2)}{\mu_P}-1\right] = E^F(-2\mu\alpha)R_p,\ W=\alpha\mu, \tag{4}$$

where μ is the reduced mass of two particles, R_p is the Zemach radius. In the coordinate representation the Zemach correction (4) is determined by the contraction of the charge

ρ_E and magnetic moment ρ_M distributions. The Zemach radius represents the integral characteristic of the proton structure effects in the hyperfine splitting of the energy levels. It may be considered as new fundamental proton parameter in the hydrogen atom. Numerical value of the Zemach contribution is equal

$$\Delta E_Z = -1.362 \pm 0.068\ meV, \tag{5}$$

where the 5% estimation of the uncertainty is connected with the measurement of the proton electromagnetic form factors [12]. So, the measurement of the muonic hydrogen hyperfine splitting as for the electronic hydrogen with similar accuracy 30 ppm as in the case of the Lamb shift can give new information about possible value of the contributions δ^{str} and δ^{pol} [19].

Such experiment demands corresponding theoretical study of different order corrections with the same precision. Analytical calculation of the hydrogen hyperfine splitting was carried out during many years [18, 20] and reached the accuracy 10^{-8}. But these calculations can not be used directly for the muonic hydrogen after the replacement the electron mass to the muon mass. The reason consists in the proton structure effects. Indeed in the case of the muonic hydrogen the dominant region of intermediate loop momenta is of order the muon mass. So, the calculation of higher order amplitudes with good accuracy can be based only on their direct integration with the account of experimental data on the proton electromagnetic form factors.

The investigation of different contributions to the energy levels of the muonic atoms was done many years ago in Ref. [21]. So, at present there is need for new more complete analysis of all possible corrections in the HFS of the μp with the declared accuracy 30 ppm. Main corrections of order α^5 to the hyperfine splitting of the $2S$ state in the μp were studied in Ref.[22]. They are very important for the extraction of the Lamb shift value $2P - 2S$ in the experiment. In this study we calculate different contributions of orders α^5 and α^6 to the muonic hydrogen HFS which are determined by the effects of electromagnetic and strong interactions. The aim of the work consists in obtaining the numerical value of the ground state HFS in the muonic hydrogen with designated accuracy which can serve as reliable guide for corresponding experiment. Some basic problems of the HFS measurement in the muonic hydrogen were discussed in Ref.[23].

II. EFFECTS OF VACUUM POLARIZATION IN THE ONE-PHOTON INTERACTION

Our calculation of different energy levels of the hydrogen-like atoms are carried out on the basis of the quasipotential approach where the two-particle bound state is described by the Schroedinger-type equation [24]:

$$\left[G^f\right]^{-1}\psi_M \equiv \left(\frac{b^2}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\psi_M(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q},M)\psi_M(\mathbf{q}), \tag{6}$$

where

$$b^2 = E_1^2 - m_1^2 = E_2^2 - m_2^2,$$

$\mu_R = E_1 E_2/M$ is the relativistic reduced mass, $M = E_1 + E_2$ is the bound state mass. The quasipotential of the equation (6) is constructed in the quantum electrodynamics by

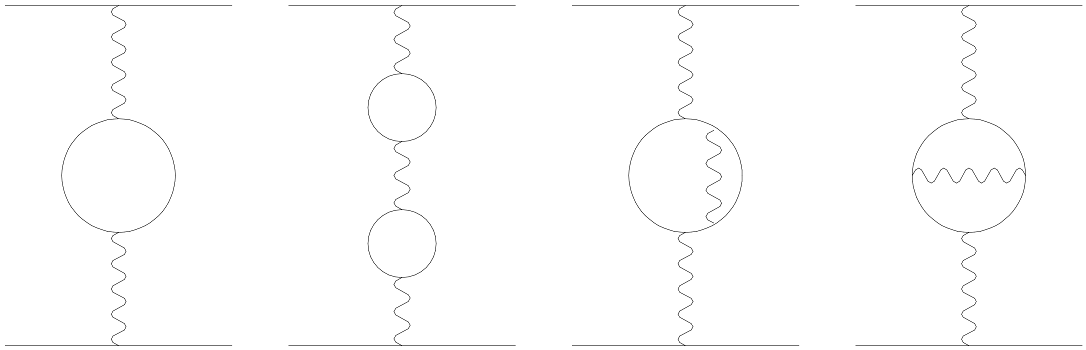


FIG. 1: Effects of the one- and two-loop vacuum polarization in the one-photon interaction.

the perturbative series using projected on positive states the two-particle off mass shell scattering amplitude T at zero relative energies of the particles:

$$V = V^{(1)} + V^{(2)} + V^{(3)} + ..., \qquad T = T^{(1)} + T^{(2)} + T^{(3)} + ..., \tag{7}$$

$$V^{(1)} = T^{(1)},\ V^{(2)} = T^{(2)} - T^{(1)} \times G^f \times T^{(1)}, ... \ . \tag{8}$$

We take the ordinary Coulomb potential as initial approximation for the quasipotential $V(\vec{p},\vec{q},M)$: $V(\vec{p},\vec{q},M) = V^C(\vec{p}-\vec{q}) + \Delta V(\vec{p},\vec{q},M)$.

The increase of the lepton mass when we change the electronic hydrogen to the muonic hydrogen leads to the decrease of the Bohr radius in the μp. As a result the electron Compton wave length and the Bohr radius are of the same order:

$$\frac{\hbar^2}{\mu e^2} : \frac{\hbar}{m_e c} = 0.737384$$

(m_e is the electron mass, μ is the reduced mass in the atom μp). An important consequence of last relation is the increase the role of the electron vacuum polarization effects in the energy spectrum of the μp [25]. The effects of the vacuum polarization in the one-photon interaction are shown in Fig.1.

To obtain the contribution of the diagram (a) Fig.1 (the electron vacuum polarization) to the interaction operator there is need to make the following substitution in the photon propagator [25]:

$$\frac{1}{k^2} \to \frac{\alpha}{\pi}\int_0^1 dv \frac{v^2\left(1-\frac{v^2}{3}\right)}{k^2(1-v^2)-4m_e^2}. \tag{9}$$

At $(-k^2) = \vec{k}^2 \sim \mu_e^2(Z\alpha)^2 \sim m_e^2(Z\alpha)^2$ (electronic hydrogen, μ_e is the reduced mass in hydrogen atom) we obtain $-\alpha/15\pi m_e^2$ omitting first term in the denominator of right part of Eq.(9). But when $\vec{k}^2 \sim \mu^2(Z\alpha)^2 \sim m_1^2(Z\alpha)^2$ (muonic hydrogen, m_1 is the muon mass) than $\mu\alpha$ and m_e are of the same order and it is impossible to use expansion over α in the denominator of Eq.(9). To construct the hyperfine part of the quasipotential in this case

(the muonic hydrogen) in the one-photon interaction we must use exact expression (9). We take into account that the appearance of the electron mass m_e in the denominator of the amplitude leads effectively to the decrease the order of the correction. It is well known that the hyperfine splitting quasipotential has the form [26]:

$$V_{1\gamma}^{HFS}(\mathbf{k}) = \frac{4\pi Z\alpha}{m_1 m_2}\frac{1+\kappa}{4}\frac{1}{\mathbf{k}^2}[(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\mathbf{k}^2 - (\boldsymbol{\sigma}_1\mathbf{k})(\boldsymbol{\sigma}_2\mathbf{k})]. \tag{10}$$

For the S-states

$$V_{1\gamma}^{HFS}(\mathbf{k}) = \frac{8\pi Z\alpha}{3m_1 m_2}\frac{\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}(1+\kappa), \tag{11}$$

κ=1.792847337(29) is the proton anomalous magnetic moment. Averaging the potential (11) over the Coulomb wave functions we obtain main contribution of order $(Z\alpha)^4$ to the HFS of the $1S$-state in the system μp (the Fermi energy):

$$E^F = \frac{8}{3}(Z\alpha)^4\frac{\mu^3}{m_1 m_2}(1+\kappa) = 182.443\ meV. \tag{12}$$

The modification of the Coulomb potential due to the vacuum polarization (VP) is determined by means of Eq.(9) in the momentum representation as follows [25]:

$$V_{VP}^{C}(\mathbf{k}) = -4\pi Z\alpha\frac{\alpha}{\pi}\int_1^\infty \frac{\sqrt{\xi^2-1}}{3\xi^4}\frac{(2\xi^2+1)}{\mathbf{k}^2+4m_e^2\xi^2}d\xi \tag{13}$$

In the coordinate representation we obtain:

$$V_{VP}^{C}(r) = \frac{\alpha}{3\pi}\int_1^\infty d\xi\frac{\sqrt{\xi^2-1}(2\xi^2+1)}{\xi^4}\left(-\frac{Z\alpha}{r}e^{-2m_e\xi r}\right). \tag{14}$$

The contribution of the electron vacuum polarization to the hyperfine splitting part of the 1γ quasipotential for the S-states can be derived in a similar way in the momentum and coordinate representations:

$$V_{1\gamma,\ VP}^{HFS}(\mathbf{k}) = \frac{4\pi Z\alpha}{m_1 m_2}\frac{(1+\kappa)}{4}\frac{2}{3}(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\mathbf{k}^2\frac{\alpha}{\pi}\int_1^\infty \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{3\xi^4(\mathbf{k}^2+4m_e^2\xi^2)}d\xi, \tag{15}$$

$$V_{1\gamma,\ VP}^{HFS}(r) = \frac{8Z\alpha(1+\kappa)}{3m_1 m_2}\frac{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)}{4}\frac{\alpha}{\pi}\int_1^\infty \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{3\xi^4}d\xi\left[\pi\delta(\mathbf{r}) - \frac{m_e^2\xi^2}{r}e^{-2m_e\xi r}\right]. \tag{16}$$

Using Eq.(16) we can obtain the electron vacuum polarization correction of order α^5 to the HFS in the μp. Taking the wave function of the $1S$-state

$$\psi_{100}(r) = \frac{W^{3/2}}{\sqrt{\pi}}e^{-Wr}, \quad W = \mu Z\alpha, \tag{17}$$

we represent this correction in the form:

$$\Delta E_{1\gamma,VP}HFS = \frac{8\mu^3(Z\alpha)^4(1+\kappa)}{3m_1 m_2}\frac{\alpha}{\pi}\frac{m_e^3}{3W^3}\int_{m_e/W}^\infty \frac{\sqrt{\frac{W^2}{m_e^2}\xi^2-1}}{\xi^4}\left(2\frac{W^2}{m_e^2}\xi^2+1\right)d\xi\ \times \tag{18}$$

$$\times \left[1 - \int_0^\infty e^{-r(\xi+1)/\xi} r dr\right] = 0.374\ meV.$$

The contribution of the muon vacuum polarization (MVP) can be found by means (16) after the substitution $m_e \to m_1$. This correction is of order α^6 due to the reason mentioned above. Numerical value is equal

$$\Delta E^{HFS}_{1\gamma,\ MVP} = E^F \frac{3}{16}\frac{\mu}{m_1} Z\alpha^2 = 0.002\ meV. \tag{19}$$

The diagrams of the two-loop electron vacuum polarization shown in Fig.1 (b,c,d) give the contributions of the same order α^6. The interaction operator corresponding to the loop after loop amplitude can be obtained using the relation (9). In the coordinate representation

$$V^{HFS}_{1\gamma,\ VP-VP}(r) = \frac{8\pi Z\alpha(1+\kappa)}{3m_1m_2}\frac{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)}{4}\left(\frac{\alpha}{\pi}\right)^2 \int_1^\infty \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{3\xi^4} d\xi \times \tag{20}$$

$$\times \int_1^\infty \frac{\sqrt{\eta^2-1}(2\eta^2+1)}{3\eta^4} d\eta \left[\delta(\mathbf{r}) - \frac{m_e^2}{\pi r(\eta^2-\xi^2)}\left(\eta^4 e^{-2m_e\eta r} - \xi^4 e^{-2m_e\xi r}\right)\right],$$

and the contribution to the energy spectrum

$$\Delta E^{HFS}_{1\gamma,\ VP-VP} = 0.001\ meV. \tag{21}$$

To calculate the contributions of the diagrams b, c in Fig.1 which are determined by the polarization operator of the second order it is necessary to make the substitution in the photon propagator [27]:

$$\frac{1}{k^2} \to \left(\frac{\alpha}{\pi}\right)^2 \int_0^1 \frac{f(v)}{4m_e^2 + k^2(1-v^2)} dv = \left(\frac{\alpha}{\pi}\right)^2 \frac{2}{3}\int_0^1 dv \frac{v}{4m_e^2+k^2(1-v^2)} \times \tag{22}$$

$$\times\Bigg\{(3-v^2)(1+v^2)\left[Li_2\left(-\frac{1-v}{1+v}\right) + 2Li_2\left(\frac{1-v}{1+v}\right) + \frac{3}{2}\ln\frac{1+v}{1-v}\ln\frac{1+v}{2} - \ln\frac{1+v}{1-v}\ln v\right] +$$

$$\left[\frac{11}{16}(3-v^2)(1+v^2) + \frac{v^4}{4}\right]\ln\frac{1+v}{1-v} + \left[\frac{3}{2}v(3-v^2)\ln\frac{1-v^2}{4} - 2v(3-v^2)\ln v\right] + \frac{3}{8}v(5-3v^2)\Bigg\}.$$

To find numerical value of this correction we write the quasipotential in the coordinate space:

$$\Delta V^{HFS}_{1\gamma,\ 2-loop\ VP}) = \frac{8\pi Z\alpha(1+\kappa)}{3m_1m_2}\left(\frac{\alpha}{\pi}\right)^2 \int_0^1 \frac{f(v)dv}{(1-v^2)}\left[\delta(\mathbf{r}) - \frac{m_e^2}{\pi r(1-v^2)} e^{-\frac{2m_e r}{\sqrt{1-v^2}}}\right]. \tag{23}$$

The potential (23) gives the contribution to the HFS in the muonic hydrogen

$$\Delta E^{HFS}_{1\gamma,\ 2-loop\ VP} = 0.002\ meV. \tag{24}$$

We calculate all contribution numerically and the results are presented with the accuracy 0.001 meV.

III. SECOND ORDER OF THE PERTURBATION THEORY

The corrections of the second order of the perturbative series in the energy spectrum are defined by the reduced Coulomb Green function (RCGF) [28]:

$$\tilde{G}_1(\mathbf{r},\mathbf{r}')=\sum_{l,m}\tilde{g}_{nl}(r,r')Y_{lm}(\mathbf{n})Y^*_{lm}(\mathbf{n}'). \tag{25}$$

The radial wave function $\tilde{g}_{nl}(r,r')$ was obtained in Ref.[28] as an expansion over the Laguerre polynomials. For the $1S$ - state

$$\tilde{g}_{10}(r,r')=-4\mu^2 Z\alpha\left(\sum_{m=2}^{\infty}\frac{L^1_{m-1}(x)L^1_{m-1}(x')}{m(m-1)}+\frac{5}{2}-\frac{x}{2}-\frac{x'}{2}\right)e^{-\frac{x+x'}{2}}, \tag{26}$$

where $x=2\mu Z\alpha r$, L^m_n are the Laguerre polynomials:

$$L^m_n(x)=\frac{e^x x^{-m}}{n!}\left(\frac{d}{dx}\right)^n\left(e^{-x}x^{n+m}\right). \tag{27}$$

Some terms of the quasipotential contain the $\delta(\vec{r})$ so we have to know the quantity $\tilde{G}_1(\vec{r},0)$. The expression for the RCGF was found in this case in Ref.[29] on the basis of the Hoestler representation for the Coulomb Green function after the subtraction the pole term:

$$\tilde{G}_{1S}(\mathbf{r},0)=\frac{Z\alpha\mu^2}{4\pi}\frac{2e^{-x/2}}{x}\left[2x(\ln x+C)+x^2-5x-2\right], \tag{28}$$

where $C=0.5772...$ is the Euler constant. The main contribution of order α^5 in the second order of the perturbation theory can be written in general form:

$$\Delta E^{HFS}_{1\ SOPT}=\sum_{n=2}^{\infty}\frac{<\psi^c_1|V^C_{VP}|\psi^c_n><\psi^c_n|\Delta V^{HFS}_{1\gamma}|\psi^c_1>}{E^c_1-E^c_n}, \tag{29}$$

where $\Delta V^{HFS}_{1\gamma}\sim\delta(\vec{r})$. Using the relations (14), (28) we can present Eq.(29) as follows:

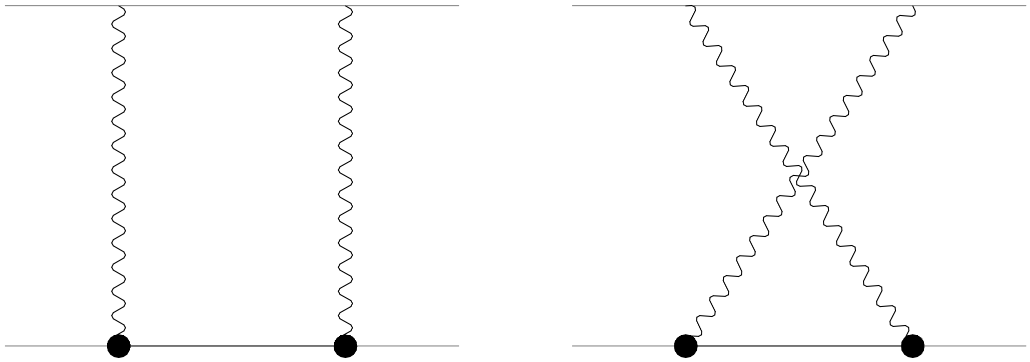
$$\Delta E^{HFS}_{1\ SOPT}=-E^F\frac{2\alpha}{3\pi}\int_1^\infty d\xi\frac{\sqrt{\xi^2-1}}{\xi^2}\left(1+\frac{1}{2\xi^2}\right)\times \tag{30}$$

$$\times\int_0^\infty dx e^{-x(1+\frac{m_e\xi}{W})}\left[2x(\ln x+C)+x^2-5x-2\right]=0.734\ meV.$$

The contribution of order α^6 in the second order of the perturbative series which is determined by the vacuum polarization can be derived from Eq.(29) changing $\Delta V^{HFS}_{1\gamma}\to\Delta V^{HFS}_{1\gamma\ VP}$. Using exact expressions for the wave function $\psi^c_1(\vec{r})$ (17) and the RCGF (28) we write this correction

$$\Delta E^{HFS}_{2\ SOPT}=-E^F\alpha^2\frac{m_e^2}{W^2}\frac{8}{9\pi^2}\int_1^\infty d\xi\left(1+\frac{1}{2\xi^2}\right)\frac{\sqrt{\xi^2-1}}{\xi^2}\times \tag{31}$$

$$\times\int_1^\infty d\eta\left(1+\frac{1}{2\eta^2}\right)\frac{\sqrt{\eta^2-1}}{\eta^2}H(\xi,\eta,\frac{m_e}{W}),$$



a b

FIG. 2: Proton structure corrections of order $(Z\alpha)^5$. Bold circle in the diagram represents the proton vertex operator.

$$H(\xi,\eta,\frac{m_e}{W})=\frac{1}{\left(1+\frac{m_e\xi}{W}\right)^2}\frac{\eta^2}{\left(1+\frac{m_e\eta}{W}\right)^2}\left[\frac{1}{\frac{W}{m_e\xi}+\frac{W}{m_e\eta}+\frac{W^2}{m_e^2\xi\eta}}-\ln\frac{\left(\frac{W}{m_e\xi}+\frac{W}{m_e\eta}+\frac{W^2}{m_e^2\xi\eta}\right)}{\left(1+\frac{W}{m_e\xi}\right)\left(1+\frac{W}{m_e\eta}\right)}\right]+ \tag{32}$$

$$+\eta^2\left[\frac{5}{2\left(1+\frac{m_e\xi}{W}\right)^2\left(1+\frac{m_e\eta}{W}\right)^2}-\frac{1}{\left(1+\frac{m_e\xi}{W}\right)^2\left(1+\frac{m_e\eta}{W}\right)^3}-\frac{1}{\left(1+\frac{m_e\xi}{W}\right)^3\left(1+\frac{m_e\eta}{W}\right)^2}\right]+$$

$$+\frac{W^2}{m_e^2}\left[\frac{1}{\left(1+\frac{m_e\xi}{W}\right)^2}\left(1-\ln\left(1+\frac{m_e\xi}{W}\right)\right)-\frac{5}{2\left(1+\frac{m_e\xi}{W}\right)^2}+\frac{1}{\left(1+\frac{m_e\xi}{W}\right)^3}-\frac{1}{\left(1+\frac{m_e\xi}{W}\right)}\right].$$

Numerical value of this contribution is equal

$$\Delta E^{HFS}_{2\ SOPT}=0.002\ meV. \tag{33}$$

The second order of the perturbative series gives also other relativistic corrections of order $(Z\alpha)^6$ including recoil effects which were studied in Ref.[30, 31, 32]. Corresponding numerical data are in the Table 1.

IV. PROTON STRUCTURE AND VACUUM POLARIZATION EFFECTS

The proton structure corrections in the system μp are relatively large in the comparison with the electronic hydrogen. In the HFS of the muonic hydrogen these corrections are defined in the leading order by the one-loop diagrams in Fig.2.

To construct the quasipotential corresponding to these diagrams we write the proton tensor:

$$M^{(p)}_{\mu\nu}=\bar u(q_2)\left[\gamma_\mu F_1+\frac{i}{2m_2}\sigma_{\mu\omega}k^\omega F_2\right]\frac{\hat p_2-\hat k+m_2}{(p_2-k)^2-m_2^2+i0}\left[\gamma_\nu F_1-\frac{i}{2m_2}\sigma_{\nu\lambda}k^\lambda F_2\right]u(p_2), \tag{34}$$

where p_2, q_2 are four momenta of the proton in initial and final states. The construction of the potential can be essentially simplified using the projection operators for the system muon-proton on the states with definite spin:

$$\hat{\pi}(^1S_0) = [u(p_2)\bar{v}(p_1)]_{S=0} = \frac{(1+\gamma^0)}{2\sqrt{2}}\gamma_5, \quad \hat{\pi}(^3S_1) = [u(p_2)\bar{v}(p_1)]_{S=1} = \frac{(1+\gamma^0)}{2\sqrt{2}}\hat{\epsilon}. \tag{35}$$

where ϵ^μ is the polarization vector of the state with the spin 1. Neglecting relative motion momenta of the particles in the initial and final states we obtain

$$\Delta E^{HFS}_{str} = E^F \frac{Z\alpha m_1 m_2}{8\pi n^3(1+\kappa)}\delta_{l0}\int \frac{id^4k}{\pi^2(k^2)^2}\left[\frac{16k^6k_0^2}{m_2^2}F_2^2 + \frac{32k^8}{m_2^2}F_2^2 - 64k^2k_0^4F_2^2 + \right. \tag{36}$$

$$\left. +16k^4k_0^2F_1^2 + 128k^4k_0^2F_1F_2 + 64k^4k_0^2F_2^2 + 32k^6F_1^2 + 64k^6F_1F_2\right]\frac{1}{(k^4-4m_1^2k_0^2)(k^4-4m_2^2k_0^2)}.$$

Transforming the integration in Eq.(36) to the Euclidean space

$$\int d^4k = 4\pi\int k^3dk\int \sin^2\phi d\phi, \quad k_0 = k\cos\phi, \tag{37}$$

we make analytical integration over the angle ϕ and present the correction (36) as one dimensional integral over the variable k:

$$\Delta E^{HFS}_{str} = -E^F\frac{Z\alpha}{8\pi n^3(1+\kappa)}\delta_{l0}\int_0^\infty \frac{dk}{k}V(k), \tag{38}$$

$$V(k) = \frac{2F_2^2k^2}{m_1m_2} + \frac{\mu}{(m_1-m_2)k(k+\sqrt{4m_1^2+k^2})}\left[-128F_1^2m_1^2 - 128F_1F_2m_1^2 + 16F_1^2k^2 + \right.$$

$$\left. +64F_1F_2k^2 + 16F_2^2k^2 + \frac{32F_2^2m_1^2k^2}{m_2^2} + \frac{4F_2^2k^4}{m_1^2} - \frac{4F_2^2k^4}{m_2^2}\right] + \frac{\mu}{(m_1-m_2)k(k+\sqrt{4m_2^2+k^2})}\times$$

$$\times\left[128F_1^2m_2^2 + 128F_1F_2m_2^2 - 16F_1^2k^2 - 64F_1F_2k^2 - 48F_2^2k^2\right].$$

To cancel infrared divergence in Eq.(38) it is necessary to add the contribution of the iteration term of the quasipotential (10) in the HFS of the μp:

$$\Delta E^{HFS}_{iter,str} = - < V_{1\gamma}\times G^f\times V_{1\gamma} >^{HFS}_{str} = -\frac{64}{3}\frac{\mu^4(Z\alpha)^5(1+\kappa)}{m_1m_2\pi n^3}\int_0^\infty \frac{dk}{k^2}, \tag{39}$$

where the angular brackets represent averaging of the interaction operator over the bound state Coulomb wave functions and the index HFS shows the hyperfine part in the interaction term of the quasipotential (10). The sum of the expressions (38) and (39) coincides with the result of Ref.[22]. The integration in Eqs.(38) and (39) was done by means of the parameterization of the proton electromagnetic form factors obtained from the analysis of elastic lepton-nucleon scattering [12]. Numerically the proton structure correction of order $(Z\alpha)^5$ is equal

$$\Delta E^{HFS}_{str} + \Delta E^{HFS}_{iter,\ str} = -1.215\ meV \tag{40}$$

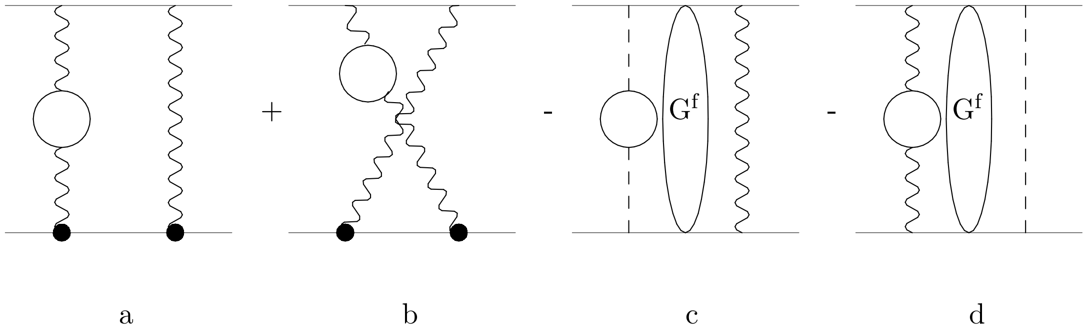


FIG. 3: Vacuum polarization and proton structure corrections of order $\alpha(Z\alpha)^5$. Dashed line in the diagram represents the Coulomb photon.

Moreover, the effects of the proton structure must be taken into account carefully in the amplitudes of higher order over α shown in Fig.3.

The contributions of the diagrams (a) and (b) in Fig.3 to the potential can be found as for the amplitudes in Fig.2. taking into account the transformation of one exchange photon propagator as in Eq.(9). Corresponding correction to the HFS of the energy level is equal

$$\Delta E^{HFS}_{str,VP}=-E^F\frac{Z\alpha}{8\pi(1+\kappa)n^3}2\frac{\alpha}{\pi}\int_0^1\frac{v^2\left(1-\frac{v^2}{3}\right)dv}{k^2(1-v^2)+4m_e^2}\int_0^\infty dkV_{VP}(k), \tag{41}$$

where the potential $V_{VP}(k)$ differs from $V(k)$ in the relation (38) only by the factor k^2. Despite of the finiteness of the integral (41) the amplitude terms of the quasipotential in Fig.3 (a), (b) must be completed by two iteration terms shown in Fig. 3 (c), (d). First addendum $< V^c \times G^f \times \Delta V^{HFS}_{VP} >$ of order $\alpha(Z\alpha)^4$ must be subtracted because the 2γ amplitudes (a) and (b) in Fig.3 produce lower order contribution. Second term $< V^c_{VP} \times G^f \times V^{HFS}_{1\gamma} >$ which is also of order $\alpha(Z\alpha)^4$ has the structure similar to Eq.(29) of the second order of the perturbative series. The contributions of discussed iteration terms to the HFS of the μp coincide:

$$\Delta E^{HFS}_{iter,VP+str}=-2< V^c\times G^f\times\Delta V^{HFS}_{VP} >^{HFS}=-2< V^c_{VP}\times G^f\times\Delta V^{HFS}_{1\gamma} >^{HFS}= \tag{42}$$

$$=-E^F\frac{4(Z\alpha)\mu\alpha}{m_e\pi^2}\int_0^\infty dk\int_0^1\frac{v^2\left(1-\frac{v^2}{3}\right)dv}{k^2(1-v^2)+1}.$$

Numerical value of the proton structure and vacuum polarization effects of the 2γ amplitudes

$$\Delta E^{HFS}_{VP,str}+2\Delta E^{HFS}_{iter,VP+str}=-0.021\ meV. \tag{43}$$

Hadronic vacuum polarization contribution to the HFS of the ground state in the μp was studied in Ref.[33]. Here we present it in the different form using the expressions (38) and (41):

$$\Delta E^{HFS}_{HVP}=-E^F\frac{\alpha(Z\alpha)}{4\pi^2(1+\kappa)}\int_{4m_\pi^2}^\infty\frac{\rho(s)ds}{k^2+s}\int_0^\infty dkV_{VP}(k). \tag{44}$$

Dividing the integration range over s on the intervals where the cross section of the e^+e^- annihilation into hadrons ($\rho(s)=\sigma^h(e^+e^-\to hadrons)/3s\sigma_{\mu\mu}$) is known from the experiment [34] we can make numerical integration in Eq.(44). The result coincides with obtained in Ref.[33]:

$$\Delta E^{HFS}_{HVP}=0.004\ meV. \tag{45}$$

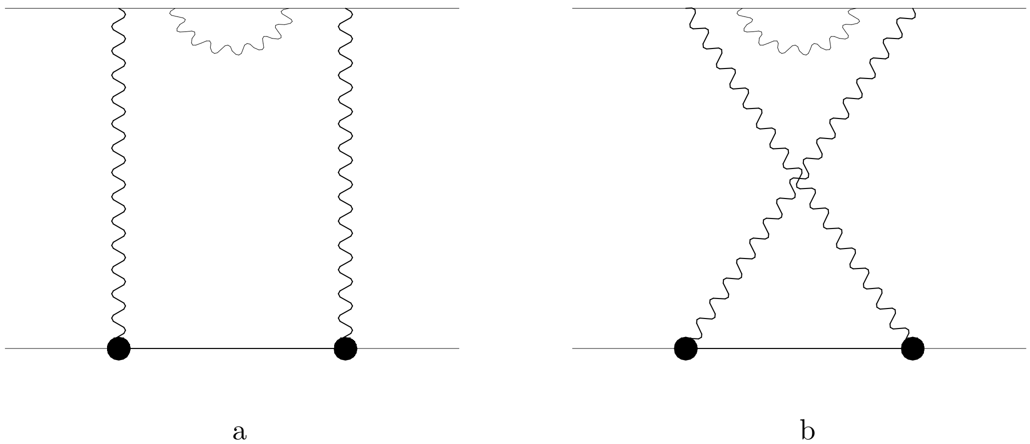


FIG. 4: Proton structure and muon self-energy effects of order $\alpha(Z\alpha)^5$.

V. PROTON STRUCTURE EFFECTS, SELF ENERGY AND VERTEX CORRECTIONS OF ORDER $\alpha(Z\alpha)^5$

There exists real number of important contributions of order α^6 which are presented in Fig.4,5. Radiative corrections of these amplitudes including recoil effects were studied earlier both in the Lamb shift and HFS of the hydrogen-like systems [18, 35, 36]. Radiative photons were taken in the Fried-Yennie (FY) gauge [37, 38, 39] where the mass shell amplitudes don't contain infrared divergences. Infrared finiteness of the Feynman diagrams in this gauge gives the possibility to make standard subtraction on the mass shell without introducing the photon mass. Let us consider radiative corrections which are determined by the self-energy insertions in the muon line. The renormalizable mass operator in the FY gauge is equal [18]:

$$\Sigma^R(p)=\frac{\alpha}{\pi}(\hat p-m)^2\int_0^1 dx\frac{-3\hat p x}{m_1^2x+(m_1^2-p^2)(1-x)}. \qquad (46)$$

Making the insertion (46) in the lepton tensor of the two-photon exchange diagrams and using the projection operators (35) we can construct the hyperfine splitting part of the quasipotential for the amplitudes in Fig. 4. In this case as before the vertex of the proton-photon interaction is determined by electric and magnetic form factors because the typical loop momenta are of order the muon mass. The contraction of the lepton and proton tensors over the Lorentz indices and the Dirac γ matrix trace calculation were made in the system Form [40]. In the Euclidean space of the variable k we can present the correction to the HFS of the muonic hydrogen as follows:

$$\Delta E^{HFS}_{2\gamma,SE}=\frac{(Z\alpha)^5\mu^3}{\pi^2n^3}\delta_{l0}\frac{\alpha}{\pi}\int_0^1 xdx\int_0^\infty kdk\int_0^\pi \sin^2\phi d\phi V_{SE}(k,\phi,x), \qquad (47)$$

$$V_{SE}(k,\phi,x)=\frac{1}{(k^2+4m_2^2\cos^2\phi)[(xm_1^2+\bar x k^2)^2+4m_1^2\bar x^2k^2\cos^2\phi]}\times \qquad (48)$$

$$\times\Bigl\{-\frac{4m_1^2}{m_2^2}k^2F_2^2(x+6\bar x)\cos^2\phi-\frac{8m_1^2}{m_2^2}k^2xF_2^2+16m_1^2F_2\cos^4\phi(4F_1\bar x-F_2x-2F_2\bar x)+$$

$$+16m_1^2\cos^2\phi(F_1^2x+6F_1^2\bar x+4F_1F_2x+8F_1F_2\bar x+F_2^2x+2F_2^2\bar x)+$$

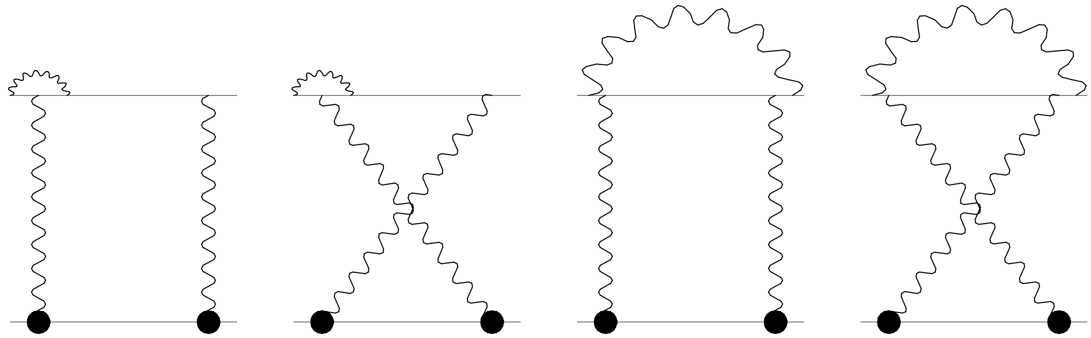


FIG. 5: Proton structure and muon vertex effects of order $\alpha(Z\alpha)^5$.

$$+32m_1^2xF_1(F_1+F_2)-\frac{4k^4}{m_2^2}F_2^2\bar x\cos^2\phi-\frac{8k^4}{m_2^2}F_2^2\bar x-16k^2F_2^2\bar x\cos^4\phi+$$
$$+16k^2\bar x\cos^2\phi(F_1^2+4F_1F_2+F_2^2)+32k^2F_1\bar x(F_1+F_2)\Biggr\}.$$

After analytical integration over the angle ϕ we present the contribution (47) in integral form which was used for numerical calculation:

$$\Delta E^{HFS}_{2\gamma,SE}=E^F\frac{m_1m_2\alpha(Z\alpha)}{\pi^2(1\kappa)n^3}\delta_{l0}\int_0^1 xdx\int_0^\infty dk\Biggl\{\left[-\frac{8F_2^2k^2}{m_2^2}+32F_1(F_1+F_2)\right]\frac{1}{h_1(k,x)}+ \quad (49)$$
$$+\left[-\frac{k^3F_2^2}{m_2^4}-\frac{6m_1^2k^3F_2^2\bar x}{m_2^4(xm_1^2+\bar xk^2)}+\frac{4k}{m_2^2}(F_1^2+4F_1F_2+F_2^2)\right]\left(\frac{1}{h_2(k,x)}-\frac{k}{h_1(k,x)}\right)+$$
$$\left[\frac{2km_1^2}{m_2^2}F_2(2F_1+F_2)\bar x-\frac{kF_2^2}{m_2^2}(xm_1^2+\bar xk^2)\right]\left[\frac{2}{h_2^2(k,x)}-\frac{k^2}{m_2^2(xm_1^2+\bar xk^2)}\left(\frac{1}{h_2(k,x)}-\frac{k}{h_1(k,x)}\right)\right],$$
$$h_1(k,x)=k\sqrt{4m_1^2\bar x^2k^2+(xm_1^2+\bar xk^2)^2}+(xm_1^2+\bar xk^2)\sqrt{4m_2^2+k^2},$$
$$h_2(k,x)=\sqrt{4m_1^2\bar x^2k^2+(xm_1^2+\bar xk^2)^2}+(xm_1^2+\bar xk^2).$$

Numerical value is equal

$$\Delta E^{HFS}_{2\gamma,SE}=0.008\ meV. \quad (50)$$

Let us consider calculation of the vertex corrections. The renormalizable expression of the one-particle vertex operator in the FY gauge was obtained in Ref. [41] ($p_1^2=m_1^2$):

$$\Lambda_\mu^R(p,p-k)=\frac{\alpha}{4\pi}\int_0^1 dx\int_0^1 dz\left[\frac{F_\mu^{(1)}}{\Delta}+\frac{F_\mu^{(2)}}{\Delta^2}\right], \quad (51)$$

where $\Delta=m_1^2x+2pk(1-x)z-k^2z(1-xz)$, the functions $F_\mu^{(1)}$, $F_\mu^{(2)}$ were determined in Ref.[41]. The lepton tensor can be divided into two parts:

$$M^{(l)(1)}_{\mu\nu}=\frac{\bar v(p_1)F_\nu^{(1)}(-\hat p_1-\hat k+m_1)\gamma_\mu v(q_1)(k^2-2k^0m_1)[m_1^2x-k^2z(1-xz)+2m_1k^2\bar x^2]}{(k^4-4k_0^2m_1^2)[(m_1^2x-k^2z(1-xz))^2-4m_1^2k_0^2\bar x^2z^2]}, \quad (52)$$

$$M_{\mu\nu}^{(l)(2)}=\frac{\bar v(p_1)F_\nu^{(2)}(-\hat p_1-\hat k+m_1)\gamma_\mu v(q_1)(k^2-2k^0m_1)[m_1^2x-k^2z(1-xz)+2m_1k^2\bar x^2]^2}{(k^4-4k_0^2m_1^2)[(m_1^2x-k^2z(1-xz))^2-4m_1^2k_0^2\bar x^2z^2]^2}. \tag{53}$$

Remaining for the simplicity the main contribution over m_1/m_2 we write this type vertex corrections as follows:

$$\Delta E^{HFS}_{2\gamma,vert\ 1}=-E^F\left(\frac{\alpha}{\pi}\right)^2\frac{8m_1m_2}{(1+\kappa)\pi n^3}\int_0^1dx\int_0^1dz\int_0\pi\sin^2\phi d\phi\int_0^\infty kdk\times \tag{54}$$

$$\times\frac{V_1(x,k,\phi)[F_1(F_1+F_2)-(1+\kappa)]}{(k^2+4m_1^2\cos^2\phi)(k^2+4m_2^2\cos^2\phi)\left[[m_1^2x+k^2z(1-zx)]^2+4m_1^2k^2\cos^2\phi\bar x^2z^2\right]},$$

$$V_1(x,k,\phi)=-2m_1^4x^2(1-x)+k^2m_1^2(6x^3z^2-8x^2z^2-3x^2z+8xz-3x)+ \tag{55}$$
$$+k^4(4x^3z^4-6x^2z^4-5x^2z^3+12xz^3-2xz^2-6z^2+3z),$$

$$\Delta E^{HFS}_{2\gamma,vert\ 2}=-E^F\left(\frac{\alpha}{\pi}\right)^2\frac{32m_1^3m_2}{(1+\kappa)\pi n^3}\int_0^1x(1-x)dx\int_0^1dz\int_0\pi\sin^2\phi d\phi\int_0^\infty k^3dk\times \tag{56}$$

$$\times\frac{V_2(x,k,\phi)F_1(F_1+F_2)}{(k^2+4m_1^2\cos^2\phi)(k^2+4m_2^2\cos^2\phi)\left[[m_1^2x+k^2z(1-zx)]^2+4m_1^2k^2\cos^2\phi\bar x^2z^2\right]^2},$$

$$V_2(x,k,\phi)=m_1^4x^2z(2z-1)-k^2m_1^2xz^2(4xz^2-2xz-4z+2)+ \tag{57}$$
$$+k^4z^3(2x^2z^3-x^2z^2-4xz^2+2xz+2z-1).$$

The iteration contribution is equal

$$\Delta E^{HFS}_{iter,\ 2\gamma\ vert}=<V_{1\gamma}\times G^f\times V_{1\gamma}>^{HFS}_{vert}=F^F\left(\frac{\alpha}{\pi}\right)^2\int_0^1dz\int_0^1dx\int_0^\infty dk\frac{4\mu}{k^2}, \tag{58}$$

After analytical integration in Eqs.(54) and (56) over the angle ϕ and subtraction (58) (one photon is the Coulomb-like and the other one contains the hyperfine part of the potential with the value of magnetic form factor at zero point) we have the expressions of the diagrams (a) and (b) in Fig. 5:

$$\Delta E^{HFS}_{2\gamma,\ vert}=-E^F\left(\frac{\alpha}{\pi}\right)^2\int_0^1dx\int_0^1dz\int_0^\infty dk\Biggl\{\frac{F_1(F_1+F_2)}{8k(1+\kappa)m_1^3m_2\bar x^2z^2}\Bigl[-2m_1^4x^2\bar x+k^2m_1^2x\times \tag{59}$$
$$\times(6x^2z^2-8xz^2-3xz+8z-3)+k^4z(4x^3z^3-6x^2z^3-5x^2z^2+12xz^2-2xz-6z-3)\Bigr]\times$$
$$\left[-\frac{\sqrt{1+b^2}}{b(a^2-b^2)(b^2-c^2)}+\frac{\sqrt{1+a^2}}{a(a^2-b^2)(a^2-c^2)}+\frac{\sqrt{1+c^2}}{c(b^2-c^2)(a^2-c^2)}\right]+\frac{F_1(F_1+F_2)x}{2(1+\kappa)m_1^3m_2k\bar x^3z^4}\times$$
$$\Bigl[m_1^4x^2z(2z-1)-2k^2m_1^2xz^2(2xz^2-xz-2z+1)+k^4z^3(2x^2z^3-x^2z^2-4xz^2+2xz+2z-1)\Bigr]\times$$
$$\times\Biggl[-\frac{\sqrt{1+b^2}}{b(a^2-b^2)(b^2-c^2)^2}+\frac{\sqrt{1+a^2}}{a(a^2-b^2)(a^2-c^2)}+\frac{1}{2c\sqrt{1+c^2}(b^2-c^2)(a^2-c^2)}-$$
$$-\frac{\sqrt{1+c^2}}{2c^3(b^2-c^2)(a^2-c^2)}+\frac{\sqrt{1+c^2}}{c(b^2-c^2)(a^2-c^2)}+\frac{\sqrt{1+c^2}}{c(b^2-c^2)(a^2-c^2)^2}\Biggr]+\frac{4\mu}{k^2}\Biggr\},$$

$$a^2=\frac{k^2}{4m_1^2},\quad b^2=\frac{k^2}{4m_2^2},\quad c^2=\frac{[m_1^2x+k^2z(1-xz)]^2}{4m_1^2k^2\bar x^2z^2}. \tag{60}$$

Numerical value of vertex correction (59) is equal

$$\Delta E^{HFS}_{2\gamma,\ vert}=-0.014\ meV \tag{61}$$

Next vertex type diagram with one rounded photon and two exchanged photons is the diagram of the "jellyfish" type. Its contribution to the energy spectrum is of order $\alpha(Z\alpha)^5$. At small loop momenta this diagram gives the finite answer in the FY gauge. The lepton tensor relating to the diagrams (c) and (d) in Fig.5 was obtained in Ref.[36]:

$$L^{(\mu)}_{\mu\nu}=\frac{\alpha}{4\pi}\int_0^1 xdx\int_0^1(1-z)dz\sum_{n=1}^3\frac{M^{(n)}_{\mu\nu}}{\Delta^n}, \tag{62}$$

where Δ has the form as in Eq.(51). The tensor functions $M^{(n)}_{\mu\nu}$ are written explicitly in Ref.[36]. The character of further transformations of the amplitudes (c), (d) in Fig.5 to construct the HFS part of the potential is the same as for other amplitudes shown in Fig.4,5. Omitting the details of such transformations which were carried out by means of analytical system Form [40] we write here three contributions to the HFS corresponding to the functions $M^{(n)}_{\mu\nu}$ in the leading order over m_1/m_2:

$$\Delta E^{HFS}_{1,\ jellyfish}=-\frac{64\alpha(Z\alpha)^5\mu^3\delta_{l0}}{\pi^3n^3}\int_0^1 xdx\int_0^1(1-z)(1-3xz)\int_0^\infty kdkF_1(F_1+F_2)\times \tag{63}$$

$$\times\int_0^\pi\frac{\sin^2\phi d\phi}{(k^2+4m_2^2\cos^2\phi)}\frac{[m_1^2x+k^2z(1-xz)]}{[m_1^2x+k^2z(1-xz)]^2+4m_1^2k^2\cos^2\phi\bar x^2z^2},$$

$$\Delta E^{HFS}_{2,\ jellyfish}=-\frac{128\alpha(Z\alpha)^5\mu^3\delta_{l0}}{3\pi^3n^3}\int_0^1 xdx\int_0^1(1-z)dz\int_0^\infty kdkF_1(F_1+F_2)\times \tag{64}$$

$$\times\int_0^\pi\frac{\sin^2\phi d\phi}{(k^2+4m_2^2\cos^2\phi)}\frac{[m_1^2x+k^2z(1-xz)]^2[k^2xz^2(1-xz)+m_1^2(x^2z+2xz-x-3z)]}{\{[m_1^2x+k^2z(1-xz)]^2+4m_1^2k^2\cos^2\phi\bar x^2z^2\}^2},$$

$$\Delta E^{HFS}_{3,\ jellyfish}=\frac{512\alpha(Z\alpha)^5\mu^3\delta_{l0}}{3\pi^3n^3}\int_0^1 xdx\int_0^1(1-z)z^2dz\int_0^\infty k^3dkm_1^2F_1(F_1+F_2)\times \tag{65}$$

$$\times(x+xz-x^2z-1)\int_0^\pi\frac{\sin^2\phi d\phi}{(k^2+4m_2^2\cos^2\phi)}\frac{[m_1^2x+k^2z(1-xz)]^3}{\{[m_1^2x+k^2z(1-xz)]^2+4m_1^2k^2\cos^2\phi\bar x^2z^2\}^3}.$$

The integration over the angle ϕ can be done in Eqs.(63)-(65) analytically. Omitting intermediate expressions we can write final numerical result to the HFS of the μp:

$$\Delta E^{HFS}_{jellyfish}=\sum_{n=1}^3\Delta E^{HFS}_{n,\ jellyfish}=0.004\ meV. \tag{66}$$

In the point-like proton approximation when the nucleus form factors entering the Feynman amplitudes in Fig. 4,5 are changed on their values at $k^2=0$ ($F_1(0)=1$, $F_2(0)=\kappa$) the contributions (63)-(65) will increase twofold.

TABLE I: Corrections of orders α^5, α^6 to the ground state HFS in the muonic hydrogen.

Contribution to HFS of μp	Numerical value in meV	Reference
The Fermi energy E^F	182.443	[18], (12)
Muon AMM correction $a_\mu E^F$ of order α^5, α^6	0.213	[18]
Relativistic correction $\frac{3}{2}(Z\alpha)^2 E^F$ of order α^6	0.015	[43]
Relativistic and radiative recoil corrections with the account proton AMM of order α^6	0.014	[30]
One-loop electron vacuum polarization contribution of 1γ interaction of order α^5	0.374	(18)
One-loop muon vacuum polarization contribution of 1γ interaction of order α^6	002	(19)
Vacuum polarization corrections of orders α^5, α^6 in the second order of perturbative series	0.736	(30)+(33)
Proton structure corrections of order α^5	-1.215	[22], (40)
Proton structure corrections of order α^6	-0.014	[8]
Electron vacuum polarization contribution+ proton structure corrections of order α^6	-0.021	(43)
Two-loop electron vacuum polarization contribution of 1γ interaction of order α^6	0.003	(21)+(24)
Muon self energy + proton structure correction of order α^6	0.008	(50)
Vertex corrections + proton structure corrections of order α^6	-0.014	(61)
"Jellyfish" diagram correction + proton structure corrections of order α^6	0.004	(66)
HVP contribution of order α^6	0.004	(45)
Proton polarizability contribution of order α^5	0.084	[16]
Weak interaction contribution	0.002	[44]
Summary contribution	182.638 ± 0.062	

VI. CONCLUSION

We made the calculation of different quantum electrodynamical effects, effects of the proton structure and polarizability, the hadron vacuum polarization to HFS of muonic hydrogen. The corrections of order α^5 and α^6 were considered. Working with the vacuum polarization diagrams we take into account that the ratio $\mu\alpha/m_e$ is very close to 1 and don't increase the order of corresponding contributions. Obtained numerical results are presented in the Table 1. We include here also QED corrections to the Fermi energy which are determined by muon anomalous magnetic moment $a_\mu E^F$ [18] (experimental value of muon anomalous magnetic moment $a_\mu^{exp} = 11659203(8) \times 10^{-10}$ [42] was used), the Breit relativistic correction of order $(Z\alpha)^6$ [43], relativistic and radiative recoil effects of the same order $(Z\alpha)^6 m_1/m_2$ with the

account of the proton anomalous magnetic moment [30], the proton structure corrections of order $(Z\alpha)^6\ln(Z\alpha)^2$ [8], the hadron vacuum polarization contribution [33] and the proton polarizability correction [16], the weak interaction contribution due to Z boson exchange [44].

Let us point out some peculiarities of this investigation.

1.The effects of the vacuum polarization play very important role in the case of the muonic hydrogen. They lead to essential modification of the spin-dependent part of the quasipotential of the one-photon interaction.

2. We took into account consistently the proton structure in the loop amplitudes by means of electromagnetic form factors. The point-like proton approximation gives essentially increased results (approximately twofold).

3. The calculation of muon self-energy and vertex corrections of order $\alpha(Z\alpha)^5$ was done on the basis of the expressions for the lepton factors in the amplitude terms of the quasipotential obtained by Eides, Grotch and Shelyuto. We supplemented these relations by the subtraction of the iteration terms of the potential.

Total value of the ground state HFS in the muonic hydrogen shown in the Table 1 can be considered as definite guide for the future experiment which is prepared [23]. Numerical values of the corrections were obtained with the accuracy 0.001 meV. Theoretical error connected with the uncertainties of fundamental physical constants (fine structure constant, the proton magnetic moment etc.) entering the Fermi energy compose the value near 10^{-5} meV. Other source of theoretical uncertainty is connected with the corrections of higher order. Its estimation can be found from the leading correction of the next order on α and m_1/m_2 in the form: $\alpha(Z\alpha)^2\ln(Z\alpha)^2/\pi\approx 0.0005$ meV (the value of fine structure constant is $\alpha^{-1}=137.03599976(50)$ [1]).

It is useful to compare the summary result for the ground state HFS in the muonic hydrogen obtained in this work (see the Table 1) with that one which can be founded in the point like proton approximation when we take into account only the values of the proton electromagnetic form factors at $k^2=0$: $G_E(0)=1$, $G_M(0)=\mu$ (with the exception of the Zemach correction). In this approximation the result of the ground state HFS may be presented with the accuracy $O((m_1/m_2)\alpha^6)$ as follows [18]:

$$\Delta E^{hfs}(QED)=E^F\Biggl\{1-2\mu\alpha R_p+\frac{3}{2}(Z\alpha)^2+a_\mu+\alpha(Z\alpha)\left(\ln 2-\frac{5}{2}\right)+ \tag{67}$$

$$+\frac{1}{1+\kappa}\Biggl[-\frac{3\alpha}{\pi}\frac{m_1m_2}{m_2^2-m_1^2}\ln\frac{m_2}{m_1}+(Z\alpha)^2\frac{\mu^2}{m_1m_2}\biggl[\left(2(1+\kappa)+\frac{7\kappa^2}{4}\right)\ln(Z\alpha)^{-1}-$$

$$-\left(8(1+\kappa)-\frac{\kappa(12-11\kappa)}{4}\right)\ln 2+3\frac{11}{18}+\frac{\kappa(11+31\kappa)}{36}\biggr]\Biggr]-\frac{2}{3}(Z\alpha)^2\ln(Z\alpha)^{-2}m_1^2r_p^2\Biggr\}=$$

$$=181.177\ meV.$$

Essential difference between this numerical value and 182.638 meV obtained in our study can be explained by some reasons: the modification of the Breit potential due to electron vacuum polarization for the muonic hydrogen, the effects of the proton structure in the two-photon and three-photon interactions, hadronic vacuum polarization and proton polarizability effects in our calculations. Further improvement of theoretical result presented in the Table 1 is connected first of all with the corrections on the proton structure and polarizability which give the theoretical error near 340 ppm. The most part of this error is

determined by the proton structure corrections of order $(Z\alpha)^5$ (the Zemach correction). So, the measurement of the hyperfine splitting of the levels $1S$ and $2S$ in the muonic hydrogen with the accuracy 30 ppm will lead to more accurate value (with relative error 10^{-3}) for the Zemach radius which than can be used for the improvement of theoretical result for the ground state hydrogen hyperfine structure and more reliable estimation of the proton polarizability effect. The increase the number of the tasks due to excited states of simple atoms [45] and the inclusion new simple atoms where the hyperfine structure of the energy spectrum is studied will decrease the uncertainties in the determination of physical fundamental parameters and increase the accuracy for the check of the Standard Model in low energy physics.

Acknowledgments

The authors are grateful to D.D. Bakalov, A.V. Borisov, V.V. Filchenkov, A.L. Kataev, I.B. Khriplovich, A.I. Studenikin for fruitful discussions. Final part of this work was made during the visit of (A.P.M.) to the Humboldt University in Berlin. He is grateful to the colleagues of the particle theory and experiment groups for warm hospitality and German Academic Exchange Service (DAAD) for financial support of the visit under stipendium. This work was supported in part by the Russian Fond for Basic Research (grant No. 04-02-16085).

---

[1] P.J.Mohr, B.N.Taylor, *Rev. Mod. Phys.* **72**, 351 (2000).

[2] Yu.L.Sokolov, *UFN* **619**, 559 (1999).

[3] F.Kottmann, F.Biraben, C.A.N.Conte et al., in: G.Cantatore (Ed.), *Quantum Electrodynamics and Physics of the Vacuum*, QED 2000 Second Workshop Proc. New York, AIP Conf. Proc. **V.564**, 2001, p.13.
R.Pohl, F.Biraben, C.A.N.Conde et al., *Hyperfine Interactions* **127**, 161 (2000).

[4] A.C.Zemach, *Phys. Rev.* **104**, 1771 (1956).

[5] F.Guerin, *Nuovo Cimento* **A50**, 1 (1967).

[6] G.M.Zinoviev, B.V.Struminskii, R.N.Faustov et al., *Yad. Fiz.* **11**, 1284 (1970).

[7] A.I.Milshtein, S.S.Petrosyan, I.B.Khriplovich, *JETP*, **109** 1146 (1996).
I.B.Khriplovich, A.I. Milshtein, *E-preprint* nucl-th/0304069, 2003.

[8] S.G.Karshenboim, *Phys. Lett.* **A225**, 97 (1997).

[9] J.L.Friar, *Can. J. Phys.* **80**, 1337 (2002).

[10] A.Pineda, *Phys. Rev.* **C67**, 025201 (2003).

[11] R.N.Faustov, A.P.Martynenko, *Phys. Rev.* **A67**, 052506 (2003).

[12] G.G.Simon, Ch.Schmitt, F.Borkowski et al., *Nucl. Phys.* **A333**, 381 (1980).

[13] S.D.Drell, J.D.Sullivan, *Phys. Rev.* **154**, 1477 (1967).

[14] J.Bernabeu, T.E.O.Ericson, *Z. Phys.* **A309**, 213 (1983).

[15] A.P.Martynenko, R.N.Faustov, *Yad. Fiz.*, **63** 915 (2000).

[16] E.V.Cherednikova, R.N.Faustov, A.P.Martynenko, *Nucl. Phys.* **A703**, 365 (2002).

[17] H. Hellwig, R.F.C.Vessot, M.W.Levine, et al, *IEEE Trans.* **IM-19**, 200 (1970).

[18] M.I.Eides, H.Grotch, V.A.Shelyuto, *Phys. Rep.* **342**, 62 (2001).

[19] D.D.Bakalov, E.Milotti, C.Rizzo et al., in: *Proc. 3rd Int. Symp. on Weak and Electromagnetic Int. in Nuclei* (WEIN-92), 656 (1992)

[20] G.T.Bodwin, D.R.Yennie, *Phys. Rev.* **D37**, 498 (1988).

[21] E.Borie, G.A.Rinker, *Rev. Mod. Phys.* **54**, 67 (1982).

[22] K.Pachucki, *Phys. Rev.* **A53**, 2092 (1996).

[23] A.Adamczak, D.Bakalov, K.Bakalova et al., *Hyperfine Interactions* **136**, 1 (2001).

[24] A.P.Martynenko, R.N.Faustov, *Teor. Mat. Fiz.* **64**, 179 (1985).

[25] V.B.Berestetskii, E.M.Lifshits, L.P.Pitaevskii, *Quantum Electrodynamics*, M., Nauka (1980).

[26] R.N.Faustov, *Fiz. Elem. Chast. i Atom. Yadra* **3**, 238 (1972).

[27] M.I.Eides, H.Grotch, V.A.Shelyuto, *Phys. Rev.* **D65**, 013003 (2001).

[28] S.A.Zapryagaev, N.L.Manakov, V.G.Palchikov, *Theory of multicharged ions with one and two electrons*, M., Energoatomizdat, 1985.

[29] V.G.Ivanov, S.G.Karshenboim, *JETP* **109**, 1219 (1996).

[30] G.T.Bodwin, D.R.Yennie, M.A.Gregorio, *Rev. Mod. Phys.* **57**, 723 (1985).

[31] T.Kinoshita, M.Nio, *Phys. Rev.* **D53**, 4909 (1996).

[32] A.P.Martynenko, R.N.Faustov, *JETP* **115**, 1221 (1999).

[33] A.P.Martynenko, R.N.Faustov, *Yad. Fiz* **61**, 471 (1998).

[34] R.R.Akhmetshin et al., *Phys. Lett.* **B476**, 33 (2000);
R.R.Akhmetshin et al., *Phys. Lett.* **B509**, 217 (2001).

[35] M.I.Eides, S.G.Karshenboim, V.A.Shelyuto, *Ann. Phys.* **205**, 231 (1991).

[36] M.I.Eides, H.Grotch, V.A.Shelyuto, *Phys. Rev.* **A63**, 052509 (2001).

[37] A.A.Abrikosov, *JETP* **30**, 96 (1956).

[38] L.P.Gorkov, *JETP* **30**, 790 (1956).

[39] H.M.Fried, D.R.Yennie, *Phys. Rev.* **112**, 1391 (1958).

[40] J.A.M.Vermaseren,*FORM*,*E-preprint* math-ph/0010025, 2000; http://www.nikhef.nl/∼form.

[41] M.I.Eides, V.A.Shelyuto, *Eur. Phys. Jour.* **C21**, 489 (2001).

[42] G.W.Bennett et al., *Phys. Rev. Lett.* **89**, 101804 (2002)

[43] G.Breit, *Phys. Rev.* **35**, 1447 (1930).

[44] M.I.Eides, *Phys. Rev.* **A53**, 2953 (1996).

[45] S.G.Karshenboim, V.G.Ivanov, *Eur. Phys. Jour.* **D19**, 13 (2002).